ERRATUM: PATTERN FORMATION BY OBLIVIOUS ASYNCHRONOUS MOBILE ROBOTS

NAO FUJINAGA, YUKIKO YAMAUCHI, HIROTAKA ONO, SHUJI KIJIMA, AND MASAFUMI YAMASHITA

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Abstract

We make a correction to a pattern formation algorithm *FORM* for oblivious asynchronous mobile robots in [N. Fujinaga, Y. Yamauchi, H. Ono, S. Kijima, and M. Yamashita, "Pattern formation by oblivious asynchronous mobile robots," *SIAM J. Comput.,* 44, 3, 740–785, 2015.]

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1 Correction

A pattern formation algorithm *FORM* in [2] for oblivious asynchronous mobile robots consists of the pattern embedding (EMB), the embedded pattern formation (FOR), the finishing (FIN) and the gathering (GAT) phases, and its correctness relies on our *incorrect* CLAIM: All robots are stationary, when they enter each of the phases for the first time. This flaw was first pointed out by Cicerone, Stefano and Navarra [1]. We modify the phases so that CLAIM will resume the correctness.

(I) Pattern Embedding Phase EMB: EMB embeds the target pattern F by transforming the initial configuration I into an ℓ-stable configuration P_ℓ, by invoking algorithm $A1$ (if $|\partial I| = 2$ and $\rho(I) = 1$), $A2$ (if $|\partial I| > 2$ and $\rho(I) = 1$), or $A3$ (if $|\partial I| > 2$ and $\rho(I) > 1$). We modify EMB so that an unstationary configuration in INV_{FIN} never emerge.

Observation 1 *There is a pattern formation algorithm $A_{n=4}$ for 4 robots, if $\rho(I)$ divides $\rho(F)$.* ∎

Proposition 1 *There is a pattern formation algorithm $A_{F=\partial F}$, if $F = \partial F$ and $\rho(I)$ divides $\rho(F)$.*

Proof (Sketch) We assume $I = \partial I$ without loss of generality, since otherwise, $A_{F=\partial F}$ can first moves all robots in $I \setminus \partial I$ to distinct positions in $C(I)$ in such a way that the symmetricity does not increase. Let $I = \partial I = \{p_1, p_2, \dots, p_n\}$ and assume that the positions p_i (of robots r_i) occur in $C(I)$ in this order counterclockwise. For $j = 1, 2, \dots, n$, let $L_j = (\ell_j, \ell_{j+1}, \dots, \ell_n, \ell_1, \dots, \ell_{j-1})$, where $\ell_j = \ell(p_j, p_{j+1})$ and $p_{n+1} = p_1$. Here $\ell(x, y)$ is the length of the arc of $C(I)$ from x to y counterclockwise. Assume without loss of generality that $L_1 \geq L_j$ in the lexicographic order, for $j = 1, 2, \dots, n$, which implies $\ell_1 \geq \ell_j$ for $j = 1, 2, \dots, n$. Observe that $L_i = L_{((i+k-1) \bmod n)+1}$ for all i, where $k = n/\rho(I)$. Let $\mathcal{R}_0 = \{r_1, r_{1+k}, \dots, r_{1+(\rho(I)-1)k}\}$. We embed $F = \partial F = \{f_1, f_2, \dots, f_n\}$ in $C(I)$ in such a way that $C(F) = C(I)$ and $p_1 = f_1$ hold, where $H_1 \geq H_j$ for $j = 1, 2, \dots, n$, $H_j = (h_j, h_{j+1}, \dots, h_n, h_1, \dots, h_{j-1})$, $h_j = \ell(f_j, f_{j+1})$, and $f_{n+1} = f_1$. Like L_j, $H_i = H_{((i+k'-1) \bmod n)+1}$ for $i = 1, 2, \dots, n$, where $k' = n/\rho(F)$. Although the embedding of F may not be unique, they are all identical since $\rho(I)$ divides $\rho(F)$. Then $A_{F=\partial F}$ moves each robot $r_j \notin R_0$ (at p_j) to f_j in $C(I)$ in such a way that all robots can continue to agree on r_1. ∎

Let $I = \{p_1, p_2, \ldots, p_n\}$ be a (general) initial configuartion, and suppose that the view of p_i is not larger than that of p_{i+1} and hence $dist(c(I), p_i) \leq dist(c(I), p_{i+1})$, for $i = 1, 2, \ldots, n-1$.

Observation 2 *Assume that $n \geq 5$ and $F \neq \partial F$. (1) If $\rho(I) = 1$, there is an initial configuration $I' = \{x_1, x_2, p_3, \ldots, p_n\}$ for some positions x_1 and x_2 ($\neq x_1$), from which an unstationary configuration in INV_{FIN} never emerge in $A1$ or $A2$. Furthermore, there is an algorithm A_{INIT} to form I' from I. (2) If $\rho(I) > 1$, an unstationary configuration in INV_{FIN} never emerge in $A3$.* ∎

Based on Observations 1 and 2 and Proposition 1, we modify EMB as follows: If $n = 4$ or $F = \partial F$, then it invokes $A_{n=4}$ or $A_{F=\partial F}$ to directly form F (and $FORM$ terminates). If $\rho(I) = 1$, EMB first invokes A_{INIT} and then $A1$ or $A2$ depending on $|\partial I|$; else if $\rho(I) > 1$, it invokes $A3$.

(II) Embedded Pattern Formation Phase FOR: FOR transforms $I \setminus \Lambda$ into $\tilde{F}$ by all robots in $I \setminus \Lambda$ invoking $CWM_{\tilde{F}}$. It may transform I into an unstationary configuration $J \in INV_{FIN}$ when $\ell \geq 2$. We modify FOR so that $\mathcal{R}$ will not reach such a J by carefully invoking CWM, provided that $\ell \geq 2$ and $F \neq \partial F$ by Proposition 1. I (resp. F) consists of $k = n/\ell$ regular ℓ-gons $I_1, I_2, \ldots, I_k$ (resp. $F_1, F_2, \ldots, F_k$) co-centered at $c(I)$ (resp. $c(F)$), where the view of (any point in) I_i (resp. F_i) is smaller than that of I_{i+1} (resp. F_{i+1}) for any $1 \leq i \leq k-1$, i.e., $\delta_{I_i} \leq \delta_{I_{i+1}}$ (resp. $\delta_{F_i} \leq \delta_{F_{i+1}}$), where δ_{I_i} (resp. δ_{F_i}) is the radius of $C(I_i)$ (resp. $C(F_i)$). Thus $\Lambda = I_k \subseteq \partial I$ and $F_k \subseteq \partial F$. By repeatedly invoking CWM, FOR moves the robots in I_i to F_i for $i = 1, 2, \ldots, k-1$ in this order. To distinguish the robots that have been moved to points in F from the others, we define the size δ_F of the embedding of F in $C(I)$ by δ_I/e, where $e \geq 1$ is the minimum integer such that $\delta_{I_i} > \delta_{F_i}$ holds for $i = 1, 2, \ldots, k$. Specifically, let i be the largest integer such that the set of points in $C(I_i)$ (excluding $C(I_i)$) form $\cup_{j=1}^{i-1} F_j$ with the correct δ_F, which can be confirmed by $\ell = |\Lambda|$, F_1 and the fact that $\cup_{j=1}^{i-1} F_j$ has been formed, although the robots in general are unaware of δ_{I_j} for any $j < i$. Then the robots in I_i move to the points in F_i by invoking CWM. We call this stage STAGE i.

Proposition 2 *Suppose that $F \neq \partial F$ and $\ell \geq 2$. Then FOR (modified) does not transform I into a configuration $J \in INV_{FIN}$, unless $n = 2\ell$ (i.e., $k = 2$) and $|\partial F| = \ell$.*

Proof (Sketch) To derive a contradiction, suppose that J emerges in FIN. Let $\zeta_F = \delta_{F_{k-1}}/\delta_{F_1}$ and $\zeta_J = d_{max}/d_{min}$, where d_{max} (resp. d_{min}) is the maximum (resp. minimum) distance of a point in $J \setminus \Lambda$ from $c(J)$. Then $\zeta_J = \zeta_F$. J emerges also in FOR. Obviously $J \notin INV_{FIN}$ until $\cup_{j=1}^{k-2} F_j$ has been formed. Since $\delta_{I_{k-1}} > \delta_{F_{k-1}}$, $\zeta_J > \zeta_F$ if J emerges in STAGE $k-1$, a contradiction. ∎

If $n = 2\ell$ and $|\partial F| = \ell$, by invoking CWM, FOR moves the robots in I_1 to F_1, where $\delta_F = \delta_I$. By the definition of CWM, FOR does not transform I into $J \in INV_{FIN}$, unless $I \in INV_{FIN}$.

(III) Finishing Phase FIN: FIN invokes $A_{\ell=1}$ or $A_{\ell>1}$. They may produce the same configuration J from different initial configurations, which may violate our assumption that $FORM$ be a function. We thus define $FORM(J) = A_{\ell>1}(J)$ if $A_{\ell=1}(J) \neq A_{\ell>1}(J)$. This correction is not sufficient enough. Whenever the robots executing $A_{\ell=1}$ reach such a J, they must be stationary to consistently switch their algorithm to $A_{\ell>1}$. We modify $A_{\ell=1}$ to satisfy this requirement: $A_{\ell=1}$ moves exactly one robot at a time. Suppose that, at a configuration, $A_{\ell=1}$ allows a robot r move along a route and its move produces a configuration $J \in INV_{A_{\ell>1}}$ at a point p en route. Then $A_{\ell=1}$ stops r at p so that J will be formed. This modification is feasible since obviously it can compute p.

(IV) Gathering Phase GAT: GAT transforms F into a pattern F^* with no multiplicities, invokes A_{F^*} to form F^*, and finally forms F from F^*. Provided the modifications in (I)–(III), A_{F^*} correctly forms F^*, and the robots can form F from F^*. Nevertheless, possibility that EMB, FOR or FIN produces an unstationary configuration $J \in INV_{GAT}$ still remains. We modify GAT as follows: Let $f \in F$ be a point with multiplicity h, and suppose that f is replaced with h points $f_1, f_2, \ldots, f_h$ in F^*. GAT orders the robots at $f_1, f_2, \ldots, f_h$ to gather at f under the following constraints: (a) Gathering at f is carried out in the increasing order of the view of f, and (b) if $f \neq c(F)$, f_i ($i = 3, 4, \ldots, h$) starts moving to $f(= f_1)$ after f_{i-1} has reached f. Provided this modification, we (re-)modify EMB so that it will not produce an unstationary configuration in INV_{GAT}. FOR and FIN do not produce such a configuration. We prove this fact only for FOR; the proof for FIN is easy.

(A) EMB: First, we extend $A_{n=4}$ and $A_{F=\partial F}$ so that they can treat a pattern F with multiplicities. Second, we modify A_{INIT} based on the following observation.

Observation 3 *Assume that $n \geq 5$ and $F \neq \partial F$. (1) If $\rho(I) = 1$, there is an initial configuration $I' = \{x_1, x_2, p_3, \ldots, p_n\}$ for some positions x_1 and x_2 ($\neq x_1$), from which an unstationary configuration in $INV_{FIN} \cup INV_{GAT}$ never emerge in A1 or A2. Furthermore, there is an algorithm A_{INIT} to form I' from I. (2) If $\rho(I) > 1$, an unstationary configuration in $INV_{FIN} \cup INV_{GAT}$ never emerge in A3.* ∎

(B) FOR: Let $f \in F$ be a point with the smallest view among those with multiplicity $h \geq 2$. FOR does not produce an unstationary configuration $J \in INV_{GAT}$ if $f \neq c(F)$. Suppose $f = c(F)$. Since ℓ, which is a divisor of $\rho(F)$, is a divisor of h, let $h = a\ell$ for some integer $a \geq 1$. Assuming that FOR produces a configuration in INV_{GAT} (to derive a contradiction), we determine the first STAGE u in which a configuration in INV_{GAT} emerges. Obviously $u \leq a$. Let J_1 and J_2 be the configuartions that STAGE u starts and ends, respectively. Then $J_1 \notin INV_{GAT}$ and $J_2 \in INV_{GAT}$, and they are identical except the difference between I_u and F_u. Since both I_u and F_u form a regular ℓ-gon, $\delta_{I_u} > \delta_{F_u}$, and $J_1 \notin INV_{GAT}$, FOR does not produce a configuration in INV_{GAT} until STAGE u ends, and stationary configuration J_2 emerges, by the definition of CWM.

References

[1] S. Cicerone, G.D. Stefano, and A. Navarra, "Asynchronous Pattern Formation: the effects of a rigorous approach," arXiv:1706.02474v1 [cs.DC] 8 Jun 2017.

[2] N. Fujinaga, Y. Yamauchi, H. Ono, S. Kijima, and M. Yamashita, "Pattern formation by oblivious asynchronous mobile robots," *SIAM J. Comput.,* 44, 3, 740–785, 2015.